Analysis of repeated measurements (KLMED8008)

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Day 3

- Practical issues …?
  - Lectures
  - Textbook
  - Software
  - Exercise 1 is now posted on the website
  - Exam
- Robust variance estimation in linear regression («sandwich»)
- A quick look at statistical models
- Variance components
  - What is «nesting»?

ROBUST («SANDWICH») VARIANCE ESTIMATION

Linear model - requirements

- Independent observations
- Residuals are Normally distributed
- Homoskedasticity

Linear model with robust standard error

- If we use the Huber-White «sandwich» estimator of the variance/covariance matrix, these assumptions may be relaxed

- Textbook:
  «By fitting an ordinary regression model with robust standard errors for clustered data instead of fitting variance-components models, we are treating the within-cluster dependence as a «nuisance», not as a phenomenon we are interested in.
  We learn nothing about the between and within-cluster variances or intraclass correlation».

```
. use winer, replace
(T4.3 -- Winer, Brown, Michels)

. regress score i.drug

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  3,    16) =    4.69
       Model |      698.2     3  232.733333           Prob > F      =  0.0155
    Residual |      793.6    16        49.6           R-squared     =  0.4680
-------------+------------------------------           Adj R-squared =  0.3683
       Total |     1491.8    19  78.5157895           Root MSE      =  7.0427

------------------------------------------------------------------------------
       score |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        drug |
          2  |        -.8   4.454211    -0.18   0.860    -10.24251    8.642507
          3  |      -10.8   4.454211    -2.42   0.028    -20.24251   -1.357493
          4  |        5.6   4.454211     1.26   0.227    -3.842507    15.04251
             |
       _cons |       26.4   3.149603     8.38   0.000     19.72314    33.07686
------------------------------------------------------------------------------

. regress score i.drug, robust cluster(person)

Linear regression                                      Number of obs =      20
                                                       F(  3,     4) =   47.85
                                                       Prob > F      =  0.0014
                                                       R-squared     =  0.4680
                                                       Root MSE      =  7.0427

                                 (Std. Err. adjusted for 5 clusters in person)
------------------------------------------------------------------------------
             |               Robust
       score |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        drug |
          2  |        -.8   1.770593    -0.45   0.675    -5.715955    4.115955
          3  |      -10.8   2.808024    -3.85   0.018    -18.59633   -3.003675
          4  |        5.6   .8154753     6.87   0.002     3.335878    7.864122
             |
       _cons |       26.4   4.270831     6.18   0.003     14.54227    38.25773
------------------------------------------------------------------------------
```

```
. pwcompare drug, bonferroni effects

Pairwise comparisons of marginal linear predictions

Margins      : asbalanced

---------------------------
             |    Number of
             |  Comparisons
-------------+-------------
        drug |            6
---------------------------

-------------------------------------------------------------------------------
             |                           Bonferroni           Bonferroni
             |   Contrast   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+-----------------------------------------------------------------
        drug |
     2 vs 1  |        -.8   4.454211    -0.18   1.000    -14.19976    12.59976
     3 vs 1  |      -10.8   4.454211    -2.42   0.165    -24.19976    2.599755
     4 vs 1  |        5.6   4.454211     1.26   1.000    -7.799755    18.99976
     3 vs 2  |        -10   4.454211    -2.25   0.235    -23.39976    3.399755
     4 vs 2  |        6.4   4.454211     1.44   1.000    -6.999755    19.79976
     4 vs 3  |       16.4   4.454211     3.68   0.012     3.000245    29.79976
-------------------------------------------------------------------------------
```

After ”robust” estimation:

```
-------------------------------------------------------------------------------
             |                           Bonferroni           Bonferroni
             |   Contrast   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+-----------------------------------------------------------------
        drug |
     2 vs 1  |        -.8   1.770593    -0.45   1.000    -9.389162    7.789162
     3 vs 1  |      -10.8   2.808024    -3.85   0.110    -24.42175    2.821749
     4 vs 1  |        5.6   .8154753     6.87   0.014     1.644122    9.555878
     3 vs 2  |        -10   2.484955    -4.02   0.095    -22.05454    2.054536
     4 vs 2  |        6.4    1.74356     3.67   0.128    -2.058022    14.85802
     4 vs 3  |       16.4   2.426932     6.76   0.015     4.626931    28.17307
-------------------------------------------------------------------------------
```

A QUICK LOOK AT STATISTICAL MODELS

One group

$y_i = \mu + \xi_i \quad \xi_i \sim N(0, \sigma^2) \quad i = 1, 2, ..., n$ (subject)

Then $y \sim N(\mu, \sigma^2)$

Hypotheses (usually)

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$

Estimators

$\hat{\mu} = \bar{y} \quad \hat{\sigma}^2 = Var(y) \ \hat{\sigma} = \sqrt{Var(y)} \qquad Var(y):$ sample variance

Example

[Day2: Vickers.dta]

$$y_i = \mu + \xi_i \quad \xi_i \sim N(0,\sigma^2) \quad i = 1, 2, ..., n \text{ (subject)}$$

Estimators

$$\hat{\mu} = \bar{y} \quad \hat{\sigma}^2 = Var(y) \ \hat{\sigma} = \sqrt{Var(y)} \qquad Var(y): \text{sample variance}$$

```
. ttest post = 60 // bare outcome, modell-eksempel

One-sample t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    post |      52    70.61538    2.697291    19.45044    65.20035    76.03042
------------------------------------------------------------------------------
    mean = mean(post)                                             t =   3.9356
Ho: mean = 60                                    degrees of freedom =       51

    Ha: mean < 60               Ha: mean != 60                 Ha: mean > 60
 Pr(T < t) = 0.9999         Pr(|T| > |t|) = 0.0003          Pr(T > t) = 0.0001
```

$\hat{\mu}$

$\hat{\sigma}$

Two independent groups

$y_{ij} = \mu_j + \xi_{ij} \quad \xi_{ij} \sim N(0, \sigma^2) \quad j = 1, 2$ (groups) $\quad i = 1, 2, \ldots, n_j$ (subject within group)

Then $y_j \sim N(\mu_j, \sigma^2)$

Hypotheses (usually)

$H_0 : \mu_1 = \mu_2 \;\leftrightarrow\; \mu_1 - \mu_2 = 0$ vs. $H_1 : \mu_1 \neq \mu_2 \;\leftrightarrow\; \mu_1 - \mu_2 \neq 0$

Estimators

$\hat{\mu}_j = \bar{y}_j \quad \hat{\sigma}^2 = Var(y_j) \; \hat{\sigma}_j = \sqrt{Var(y_j)}$

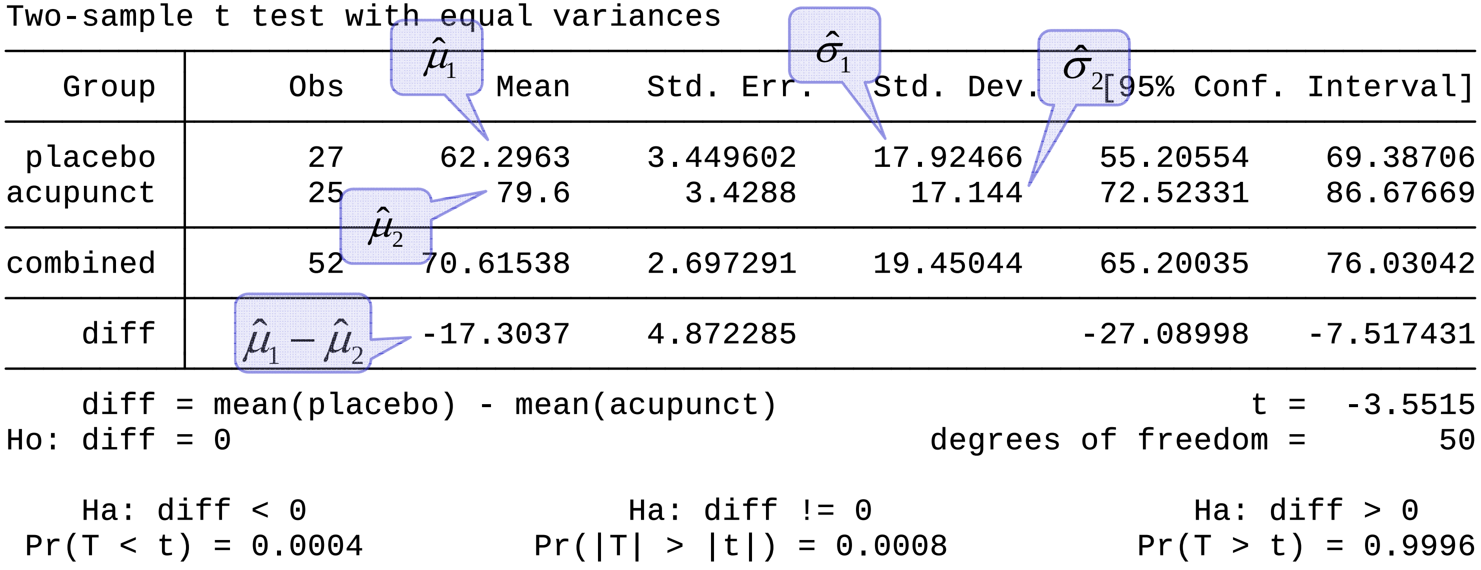
Example

[Day2: Vickers.dta]

$$y_{ij} = \mu_j + \xi_{ij} \quad \xi_{ij} \sim N(0,\sigma^2) \quad j = \text{ 1, 2 (groups)} \quad i = \text{ 1, 2, ..., } n_j \text{ (subject within group)}$$

Estimators

$$\hat{\mu}_j = \bar{y}_j \quad \hat{\sigma}^2 = Var(y_j) \; \hat{\sigma}_j = \sqrt{Var(y_j)}$$



```
. ttest post, by (group) // bare outcome

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
 placebo |      27     62.2963    3.449602    17.92466    55.20554    69.38706
acupunct |      25        79.6      3.4288      17.144    72.52331    86.67669
---------+--------------------------------------------------------------------
combined |      52    70.61538    2.697291    19.45044    65.20035    76.03042
---------+--------------------------------------------------------------------
    diff |            -17.3037    4.872285               -27.08998   -7.517431
------------------------------------------------------------------------------
    diff = mean(placebo) - mean(acupunct)                         t =  -3.5515
Ho: diff = 0                                     degrees of freedom =       50

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0004         Pr(|T| > |t|) = 0.0008          Pr(T > t) = 0.9996
```

ANOVA (more than two groups)

$y_{ij} = \mu_j + \xi_{ij} \quad \xi_{ij} \sim N(0, \sigma^2) \quad j =$ 1, 2, 3, ..., m (groups)

$i =$ 1, 2, ... , n_j subject within group

Then $y_j \sim N(\mu_j, \sigma^2)$

Hypotheses (usually)

$H_0 : \mu_1 = \mu_2 = \ldots = \mu_j$ vs. H_1 : At least one μ_j differs from the other

Estimators

$\hat{\mu}_j = \bar{y}_j \quad \hat{\sigma}^2 = MSE \;\; \hat{\sigma} = \sqrt{MSE}$

Example

[Day2: winer.dta]

```
. anova score drug

                           Number of obs =      20     R-squared     =  0.4680
                           Root MSE      = 7.04273     Adj R-squared =  0.3683

                  Source |  Partial SS    df       MS           F     Prob > F
              -----------+----------------------------------------------------
                   Model |      698.2      3   232.733333      4.69     0.0155
                         |
                    drug |      698.2      3   232.733333      4.69     0.0155
                         |
                Residual |      793.6     16         49.6
              -----------+----------------------------------------------------
                   Total |     1491.8     19   78.5157895
```

$\hat{\sigma}$

$\hat{\sigma}^2$

```
. margins drug

Adjusted predictions                              Number of obs    =         20

Expression   : Linear prediction, predict()

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        drug |
          1  |       26.4   3.149603     8.38   0.000     20.22689    32.57311
          2  |       25.6   3.149603     8.13   0.000     19.42689    31.77311
          3  |       15.6   3.149603     4.95   0.000     9.426891    21.77311
          4  |         32   3.149603    10.16   0.000     25.82689    38.17311
------------------------------------------------------------------------------
```

$\hat{\mu}_1$

$\hat{\mu}_2$

$\hat{\mu}_3$

$\hat{\mu}_4$

Example

[Day2: Vickers.dta]

$$y_i = \beta_0 + \beta_1 x_1 + \xi_i$$

$i = 1,\ 2,\ \ldots,\ n$ (subjects)

X_1: continuous predictor variable

```
. regress post pre

      Source |       SS       df       MS              Number of obs =      52
-------------+------------------------------           F(  1,    50) =   24.37
       Model |  6323.33822     1  6323.33822           Prob > F      =  0.0000
    Residual |  12970.9695    50  259.419389           R-squared     =  0.3277
-------------+------------------------------           Adj R-squared =  0.3143
       Total |  19294.3077    51  378.319759           Root MSE      =  16.107

------------------------------------------------------------------------------
        post |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         pre |   .8252027   .1671432     4.94   0.000     .4894858     1.16092
       _cons |   23.54709    9.79174     2.40   0.020       3.8798    43.21438
------------------------------------------------------------------------------
```

$\hat{\sigma}^2$

$\hat{\sigma}$

$\hat{\beta}_1$

$\hat{\beta}_0$

Example

[Day2: Vickers.dta]

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \xi_i$$

$i = 1,\ 2,\ \ldots,\ n$ (subjects)

X_1: continuous predictor variable

X_2: categorical predictor variable $(X_2 = 0,1)$

```
. regress post pre i.group // rett metode

      Source |       SS       df       MS              Number of obs =      52
-------------+------------------------------           F(  2,    49) =   18.48
       Model |  8296.11243     2  4148.05621           Prob > F      =  0.0000
    Residual |  10998.1953    49  224.452965           R-squared     =  0.4300
-------------+------------------------------           Adj R-squared =  0.4067
       Total |  19294.3077    51  378.319759           Root MSE      =  14.982

------------------------------------------------------------------------------
        post |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         pre |   .7102148   .1602363     4.43   0.000     .3882079    1.032222
     1.group |   12.70572   4.285715     2.96   0.005     4.093254    21.31819
       _cons |   23.99731   9.109231     2.63   0.011     5.691621    42.30299
------------------------------------------------------------------------------
```

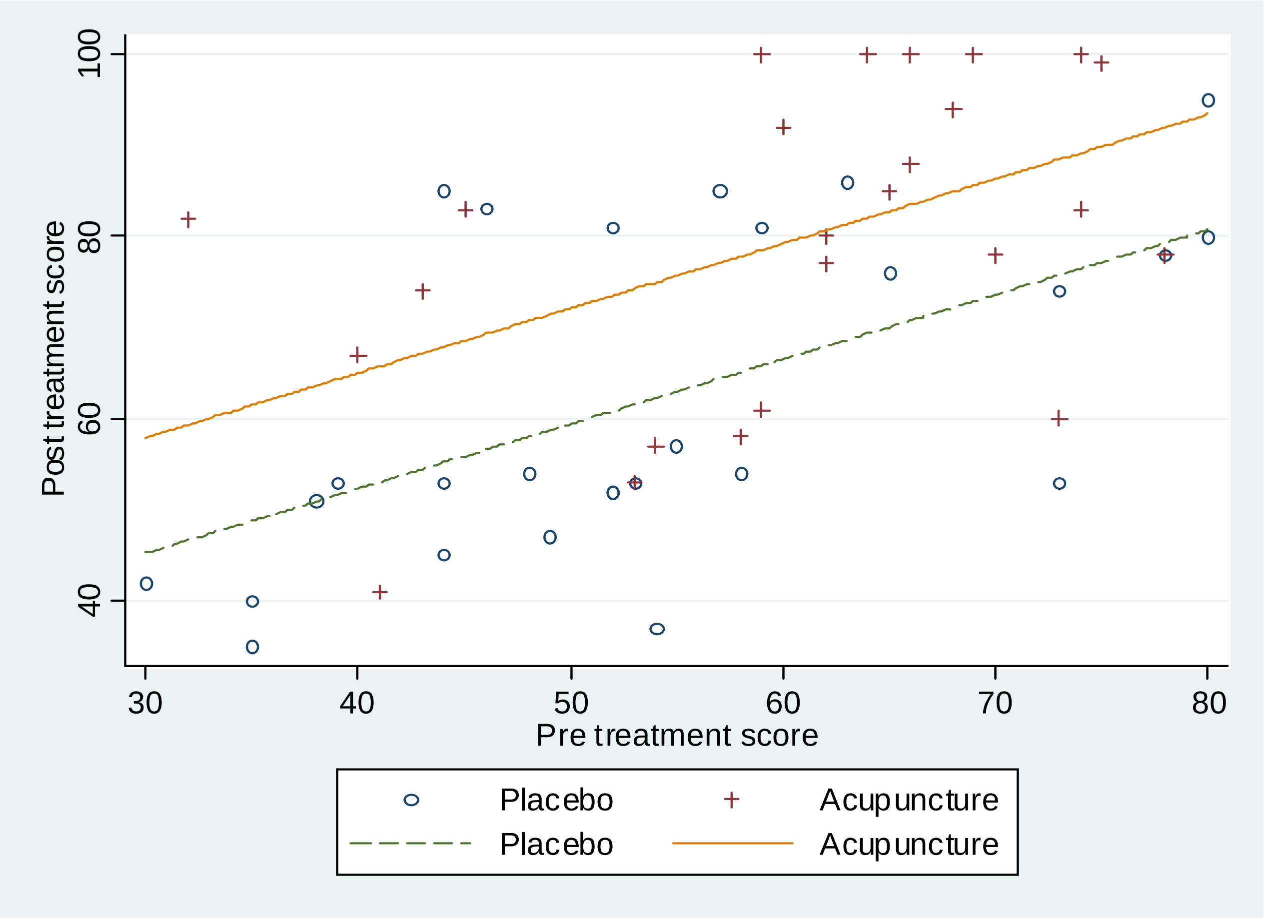
$\hat{\sigma}^2$

$\hat{\sigma}$

$\hat{\beta}_1$

$\hat{\beta}_2$

$\hat{\beta}_0$



$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \xi_i$$

$i = 1,\ 2,\ \ldots,\ n$ (subjects)

X_1: continuous predictor variable

X_2: categorical predictor variable $(X_2 = 0,1)$

VARIANCE COMPONENTS

Example: PEFR

```
. use http://www.stata-press.com/data/mlmus3/pefr, clear

. list

     +-------------------------------+
     | id   wp1   wp2   wm1   wm2 |
     |-------------------------------|
  1. |  1   494   490   512   525 |
  2. |  2   395   397   430   415 |
  3. |  3   516   512   520   508 |
  4. |  4   434   401   428   444 |
  5. |  5   476   470   500   500 |
     |-------------------------------|
  6. |  6   557   611   600   625 |
  7. |  7   413   415   364   460 |
  8. |  8   442   431   380   390 |
  9. |  9   650   638   658   642 |
 10. | 10   433   429   445   432 |
     |-------------------------------|
 11. | 11   417   420   432   420 |
 12. | 12   656   633   626   605 |
 13. | 13   267   275   260   227 |
 14. | 14   478   492   477   467 |
 15. | 15   178   165   259   268 |
     |-------------------------------|
 16. | 16   423   372   350   370 |
 17. | 17   427   421   451   443 |
     +-------------------------------+
```

Lung function (peak expiratory flow)

- Two different instruments:
  - Mini Wright peak flow meter (”wm”)
  - Wright peak flow meter (”wp)
- Measured in 17 subjects (”id”), on 2 occasions using both instruments (wp1 and wp2, and wm1 and wm2, respectively)
- We will model wm1 and wm2

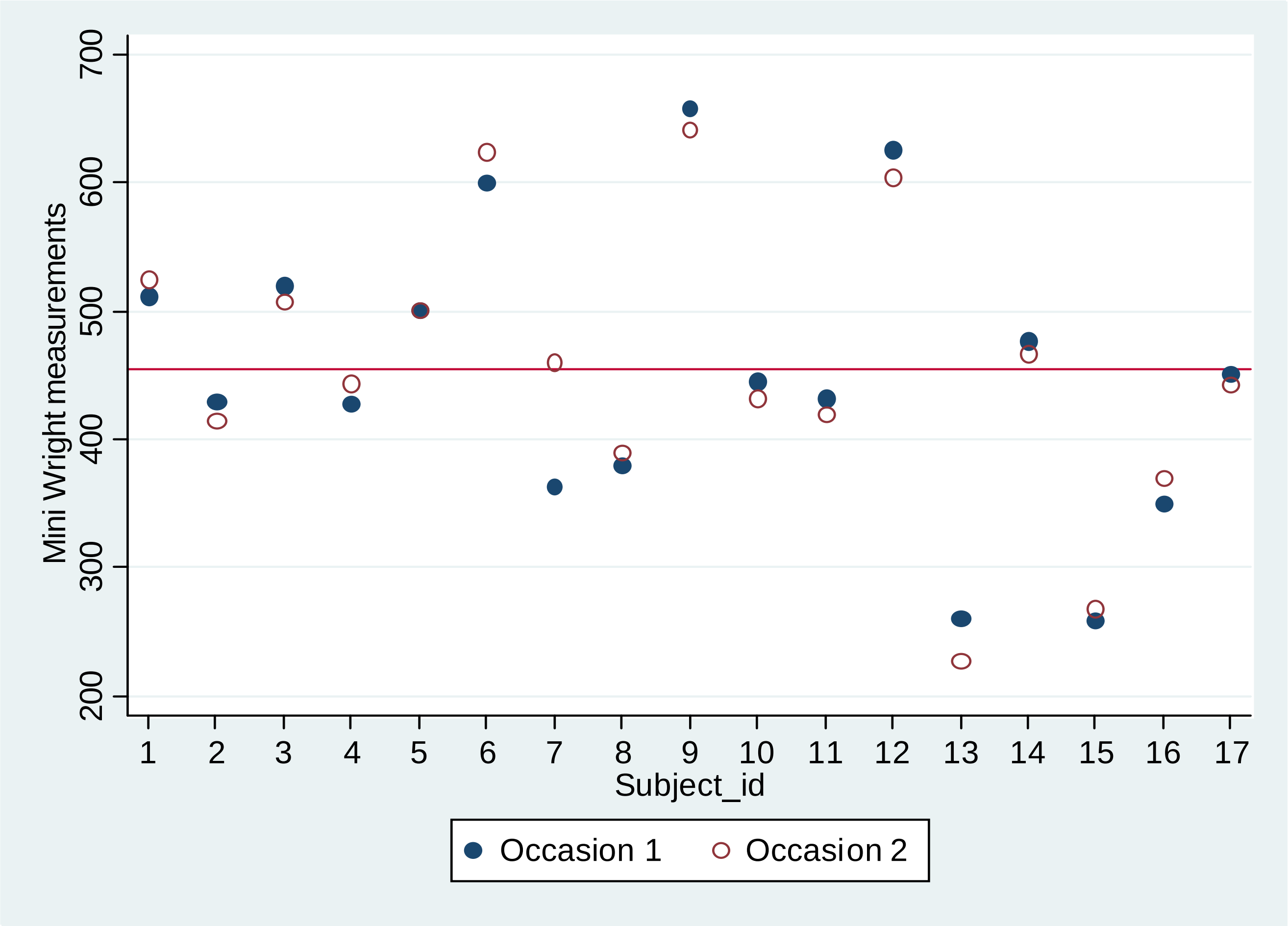
```
. gen mean_wm = (wm1 + wm2)/2 // slide 21, very simple

. egen mean_wm2 =  rowmean (wm1 wm2) // using "egen"

. summarize // slide 21

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
          id |         17           9    5.049752          1         17
         wp1 |         17    450.3529    116.3126        178        656
         wp2 |         17    445.4118    119.6129        165        638
         wm1 |         17    452.4706    113.1151        259        658
         wm2 |         17    455.3529    111.3249        227        642
-------------+---------------------------------------------------------
     mean_wm |         17    453.9118    111.2912      243.5        650
    mean_wm2 |         17    453.9118    111.2912      243.5        650
```

```
twoway (scatter wm1 id, msymbol(circle)) ///
(scatter wm2 id, msymbol (circle_hollow)), ///
xtitle (Subject_id) xlabel(1/17) ///
ytitle (Mini Wright measurements) ///
legend(order(1 "Occasion 1" 2 "Occasion 2")) ///
yline (453.9118)
```



Variance components

- Total variance is considered to be the sum of (two) independent components:
  - Between subjects
  - Within subjects (i.e. the residual)
- One aspect:
  Is variation <u>between</u> subjects larger than the variation of repeated measurements <u>within</u> the same subjects (i.e. on different occasions)?
- No particular interest in the measurement on the subject itself (other than for the subject, of course)
  - ordinary fixed-effects ANOVA is uninteresting

Variance components

$$y_{ij} = \beta + \underbrace{\varsigma_j + \varepsilon_{ij}}_{\xi_{ij}}$$

$i =$ occasions nested within subject (here: $i = 1, 2$)

$j =$ subject (here: $j =$ 1,2, ...,17)

$\varsigma_j \sim N(0,\psi)$ ψ: between-subject variance; SD(ς_j)=$\sqrt{\psi}$

Stata *xtreg*: sigma_u

$\varepsilon_i \sim N(0,\theta)$ θ: within-subject (residual) variance; SD(ε_i)=$\sqrt{\theta}$

Stata *xtreg*: sigma_e

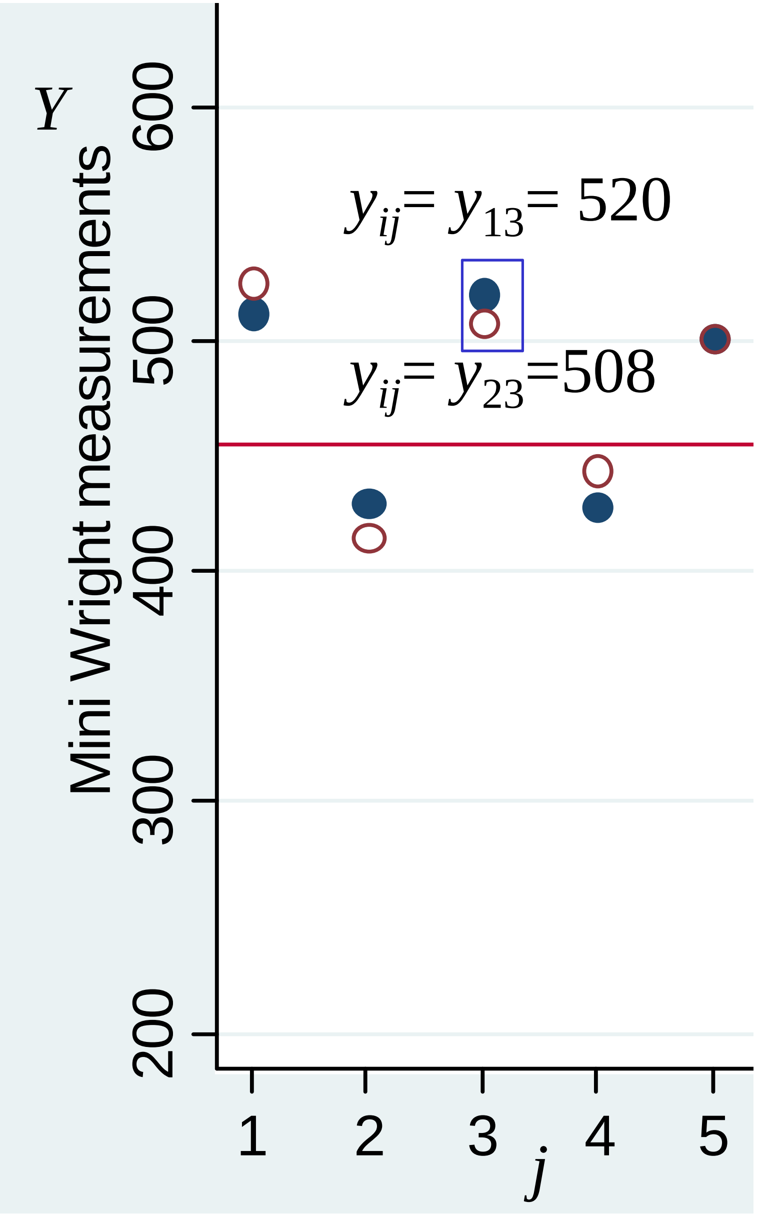
Stata *xtmixed* uses different notation:

i indicate clusters (here: subjects)

j indicate residuals (here occasions)

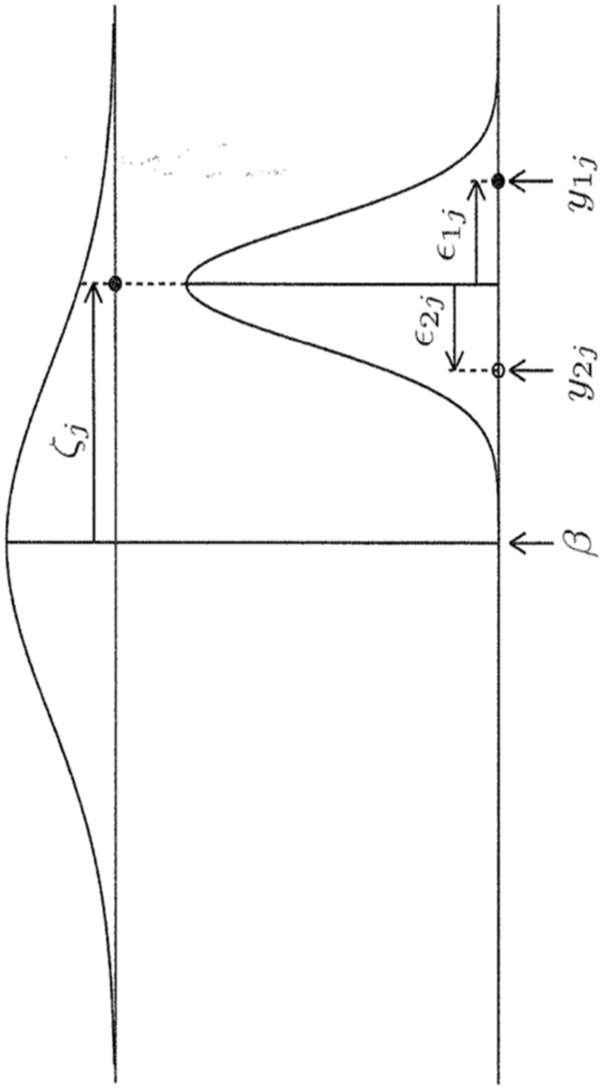
```
. list wm1 wm2 in 1/5

     +-----------+
     | wm1   wm2 |
     |-----------|
  1. | 512   525 |
  2. | 430   415 |
  3. | 520   508 |
  4. | 428   444 |
  5. | 500   500 |
     +-----------+
```



$$\hat{\beta} = 453.9$$

$$\hat{\zeta}_3 = \frac{508 + 520}{2} - 453.9$$
$$= 514 - 453.9 = 60.1$$



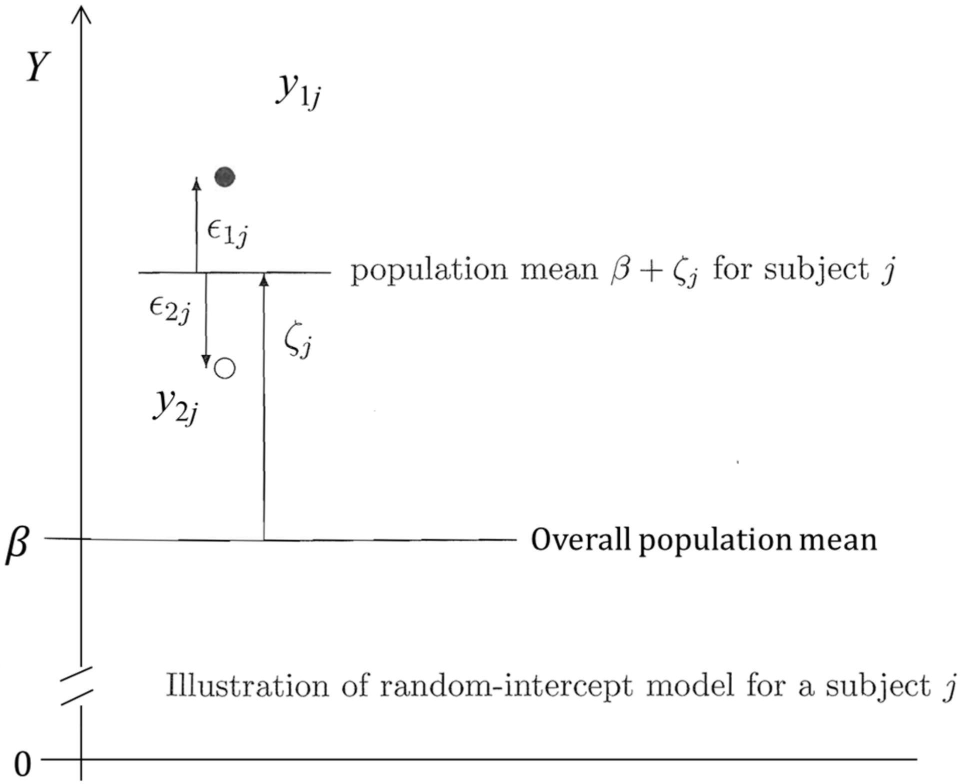


Illustration of random-intercept model for a subject j

Textbook ; figures 2.3 and 2.7 combined

Reshape from <u>wide</u> to <u>long</u> format

(usually necessary to run regression models)

```
. list

     +-------------------+
     | id   wm1      wm2 |
     |-------------------|
  1. |  1   512      525 |
  2. |  2   430      415 |
  3. |  3   520      508 |
  4. |  4   428      444 |
  5. |  5   500      500 |
     |-------------------|
  6. |  6   600      625 |
  7. |  7   364      460 |
  8. |  8   380      390 |
  9. |  9   658      642 |
 10. | 10   445      432 |
     |-------------------|
 11. | 11   432      420 |
 12. | 12   626      605 |
 13. | 13   260      227 |
 14. | 14   477      467 |
 15. | 15   259      268 |
     |-------------------|
 16. | 16   350      370 |
 17. | 17   451      443 |
     +-------------------+
```

```
. reshape long wm, i(id) j(occ) // slide 33
(note: j = 1 2)

Data                                  wide   ->   long
-----------------------------------------------------------------------------
Number of obs.                          17   ->      34
Number of variables                      3   ->       3
j variable (2 values)                        ->   occ
xij variables:
                                   wm1 wm2   ->   wm
-----------------------------------------------------------------------------
```

```
. list  //slide 33

     +----------------+
     | id   occ    wm |
     |----------------|
  1. |  1     1   512 |
  2. |  1     2   525 |
  3. |  2     1   430 |
  4. |  2     2   415 |
  5. |  3     1   520 |
     |----------------|
  6. |  3     2   508 |
  7. |  4     1   428 |
  8. |  4     2   444 |
  9. |  5     1   500 |
 10. |  5     2   500 |
     |----------------|
 11. |  6     1   600 |
 12. |  6     2   625 |
 13. |  7     1   364 |
 14. |  7     2   460 |
 15. |  8     1   380 |
     |----------------|
 16. |  8     2   390 |
```

```
. xtreg wm, i(id)

Random-effects GLS regression                   Number of obs      =        34
Group variable: id                              Number of groups   =        17

R-sq:  within  = 0.0000                         Obs per group: min =         2
       between = 0.0000                                        avg =       2.0
       overall = 0.0000                                        max =         2

                                                Wald chi2(0)       =         .
corr(u_i, X)   = 0 (assumed)                    Prob > chi2        =         .

------------------------------------------------------------------------------
          wm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   453.9118   26.99208    16.82   0.000     401.0083    506.8153
-------------+----------------------------------------------------------------
     sigma_u |  110.39704
     sigma_e |  19.910831
         rho |  .96849628   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

$\hat{\beta}$

$\sqrt{\hat{\psi}}$

$\sqrt{\hat{\theta}}$

$\widehat{ICC} = \hat{\rho}$

```
. xtreg wm, i(id) mle
Iteration 0:   log likelihood = -187.89003
Iteration 1:   log likelihood = -184.95979
Iteration 2:   log likelihood = -184.76189
Iteration 3:   log likelihood =  -184.5855
Iteration 4:   log likelihood =  -184.5784
Iteration 5:   log likelihood = -184.57839

Random-effects ML regression                    Number of obs      =        34
Group variable: id                              Number of groups   =        17

Random effects u_i ~ Gaussian                   Obs per group: min =         2
                                                               avg =       2.0
                                                               max =         2

                                                Wald chi2(0)       =      0.00
Log likelihood  = -184.57839                    Prob > chi2        =         .

------------------------------------------------------------------------------
          wm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   453.9118   26.18616    17.33   0.000     402.5878    505.2357
-------------+----------------------------------------------------------------
    /sigma_u |   107.0464   18.67858                       76.0406    150.6949
    /sigma_e |   19.91083   3.414659                       14.2269     27.8656
         rho |   .9665602   .0159494                      .9210943    .9878545
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01)=   46.27 Prob>=chibar2 = 0.000
```

$\hat{\beta}$

$\sqrt{\hat{\psi}}$

$\sqrt{\hat{\theta}}$

$ICC = \hat{\rho}$

Intraclass correlation – ICC

- ICC may take on values [0,1]
- A measure of the extent the observations within a cluster are closer than the clusters are

Example, pefr.dta

$$\mathrm{ICC} = \rho = \frac{\psi}{\psi + \theta}$$

$$\hat{\rho} = \frac{\hat{\psi}}{\hat{\psi} + \hat{\theta}} = \frac{107.05^2}{107.5^2 + 19.91^2} = 0.97$$

General tool in Stata for mixed models[1]: xtmixed

[1] ”mixed” means that both fixed effects and random effects are present in the model

```
. xtmixed wm ||id:, mle

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood = -184.57839
Iteration 1:   log likelihood = -184.57839

Computing standard errors:

Mixed-effects ML regression                     Number of obs      =        34
Group variable: id                              Number of groups   =        17

                                                Obs per group: min =         2
                                                               avg =       2.0
                                                               max =         2


                                                Wald chi2(0)       =         .
Log likelihood = -184.57839                     Prob > chi2        =         .

------------------------------------------------------------------------------
          wm |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   453.9118   26.18617    17.33   0.000     402.5878    505.2357
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
id: Identity                 |
                   sd(_cons) |   107.0464   18.67858      76.04062     150.695
-----------------------------+------------------------------------------------
                sd(Residual) |   19.91083   3.414678      14.22687    27.86564
------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =    46.27 Prob >= chibar2 = 0.0000
```

$\hat{\beta}$

$\sqrt{\hat{\psi}}$

$\sqrt{\hat{\theta}}$

Obtaining predictions of the random effects (ζ_j)

```
. predict u, reffects  // slide 38

. list id u  // slide 38

     +---------------+
     | id          u |
     |---------------|
  1. |  1   63.48996 |
  2. |  1   63.48996 |
  3. |  2  -30.87763 |
  4. |  2  -30.87763 |
  5. |  3   59.06648 |
     |---------------|
  6. |  3   59.06648 |
  7. |  4  -17.60719 |
  8. |  4  -17.60719 |
  9. |  5   45.30454 |
 10. |  5   45.30454 |
     |---------------|
 11. |  6   155.8916 |
 12. |  6   155.8916 |
 13. |  7  -41.19909 |
 14. |  7  -41.19909 |
 15. |  8  -67.73997 |
     |---------------|
 16. |  8  -67.73997 |
 17. |  9   192.7539 |
 18. |  9   192.7539 |
 19. | 10   -15.1497 |
 20. | 10   -15.1497 |
     |---------------|
 21. | 11  -27.43715 |
 22. | 11  -27.43715 |
 23. | 12   158.8406 |
 24. | 12   158.8406 |
 25. | 13  -206.8339 |
     |---------------|
```